

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Level - 1	Daily Tutorial Sheet - 1
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1. $\cos^2 \theta + \sec^2 \theta = \cos^2 \theta + \frac{1}{\cos^2 \theta} - 2 + 2 = \left(\cos \theta - \frac{1}{\cos \theta} \right)^2 + 2 = (\text{a number} \geq 0) + 2 = [\text{Always} \geq 2]$

2. $7 (\operatorname{cosec} \theta - 1) = 3 \cot \theta \Rightarrow 7 (\operatorname{cosec}^2 \theta - 1) = 3 \cot \theta (\operatorname{cosec} \theta + 1)$
 $7 \cot^2 \theta = 3 \cot \theta (\operatorname{cosec} \theta + 1) \Rightarrow 7 \cot \theta = 3 (\operatorname{cosec} \theta + 1)$

3. L.H.S : $\frac{1 + (1 + \cot^2 \alpha) \times \frac{\sin^2 \theta}{\cos^2 \theta}}{1 + (1 + \cot^2 \beta) \times \frac{\sin^2 \theta}{\cos^2 \theta}} = \frac{\cos^2 \theta + \sin^2 \theta + \cot^2 \alpha \cdot \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta + \cot^2 \beta \cdot \sin^2 \theta} = \frac{1 + \cot^2 \alpha \sin^2 \theta}{1 + \cot^2 \beta \sin^2 \theta} = \text{R.H.S.}$

Alternate solution:

$$a \cos \alpha + b \sin \alpha = c = a \cos \beta + b \sin \beta$$

$$\Rightarrow a(\cos \alpha - \cos \beta) = b(\sin \beta - \sin \alpha)$$

$$\Rightarrow 2a \sin \left(\frac{\beta - \alpha}{2} \right) \sin \left(\frac{\alpha + \beta}{2} \right) = 2b \sin \left(\frac{\beta - \alpha}{2} \right) \cos \left(\frac{\alpha + \beta}{2} \right)$$

$$\Rightarrow \tan \left(\frac{\alpha + \beta}{2} \right) = \frac{b}{a}$$

$$\Rightarrow \cos(\alpha + \beta) = \frac{1 - b^2 / a^2}{1 + b^2 / a^2} = \frac{a^2 - b^2}{a^2 + b^2}$$

4. (a) $\left. \begin{array}{l} \cos A = n \cos B \\ \sin A = m \sin B \end{array} \right\} \rightarrow \text{square and add to get, } \cos^2 A + \sin^2 A = n^2 \cos^2 B + m^2 \sin^2 B \Rightarrow 1 - n^2 = (m^2 - n^2) \sin^2 B$

(b) $\left. \begin{array}{l} \cos B = \frac{\cos A}{n} \\ \sin B = \frac{\sin A}{m} \end{array} \right\} \rightarrow \text{square and add to get, } \cos^2 B + \sin^2 B = \frac{\cos^2 A}{n^2} + \frac{\sin^2 A}{m^2}$

$$\Rightarrow 1 = \frac{1}{n^2} - \frac{\sin^2 A}{n^2} + \frac{\sin^2 A}{m^2} \Rightarrow m^2 (n^2 - 1) = (n^2 - m^2) \sin^2 A$$

5. $\tan \theta = \frac{x \sin \phi}{1 - x \cos \phi} \Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{x \sin \phi}{1 - x \cos \phi}$

$$\Rightarrow \sin \theta - x \sin \theta \cos \phi = x \sin \phi \cos \theta \Rightarrow x = \frac{\sin \theta}{\sin(\theta + \phi)} \text{ and similarly } y = \frac{\sin \phi}{\sin(\theta + \phi)}$$

$$\Rightarrow \frac{x}{y} = \frac{\sin \theta}{\sin \phi} \Rightarrow x \sin \phi = y \sin \theta$$

6. Consider, $a \cos \theta + b \sin \theta = c \Rightarrow a \cos \theta = c - b \sin \theta$
 $\Rightarrow a^2 \cos^2 \theta = (c - b \sin \theta)^2 \Rightarrow a^2 (1 - \sin^2 \theta) = c^2 + b^2 \sin^2 \theta - 2bc \sin \theta$
 $\Rightarrow (a^2 + b^2) \sin^2 \theta - 2bc \sin \theta + c^2 - a^2 = 0$

As 'α' and 'β' are value of θ as given:

$$\Rightarrow \text{Roots of above equation are } \sin \alpha \text{ and } \sin \beta \quad \Rightarrow \quad \sin \alpha \cdot \sin \beta = \frac{c^2 - a^2}{a^2 + b^2}$$

and similarly : $(a^2 + b^2) \cos^2 \theta - 2ac \cos \theta + c^2 - b^2 = 0$ [Convert the given equation in terms of $\cos \theta$]

$$\Rightarrow \text{Roots of above equation are } \cos \alpha \text{ and } \cos \beta \quad \Rightarrow \quad \cos \alpha \cdot \cos \beta = \frac{c^2 - b^2}{a^2 + b^2}$$

$$\text{Now, } \cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta = \frac{c^2 - b^2}{a^2 + b^2} - \frac{c^2 - a^2}{a^2 + b^2} = \frac{a^2 - b^2}{a^2 + b^2}$$

7. (i) Square and Subtract the two equations to get: $x^4 - y^4 = b^2 - a^2$

$$(ii) \quad \sec \theta + \operatorname{cosec} \theta = n \quad \Rightarrow \quad \sin \theta + \cos \theta = n \sin \theta \cdot \cos \theta \quad \dots (i)$$

$$\text{and } m = n \sin \theta \cdot \cos \theta \quad [\text{Using other equation}]$$

$$\Rightarrow \quad \sin \theta \cdot \cos \theta = \frac{m}{n}$$

$$(\sin \theta + \cos \theta)^2 = m \Rightarrow 1 + \frac{2m}{n} = m^2$$

$$8. \quad \sin \alpha \cdot \sin \beta - \cos \alpha \cdot \cos \beta + 1 = 0$$

$$\Rightarrow \quad \cos(\alpha + \beta) = 1 \quad \Rightarrow \quad \sin(\alpha + \beta) = 0$$

$$9. \quad \alpha = A + B \text{ and } x = A - B$$

$$\text{Also, } \frac{\tan A}{\tan B} = k \Rightarrow \frac{\sin A \cos B}{\cos A \sin B} = \frac{k}{1}$$

$$\text{Apply 'C' and 'D' to get : } \frac{\sin A \cos B - \cos A \cdot \sin B}{\sin A \cos B + \cos A \sin B} = \frac{k - 1}{k + 1}$$

$$\Rightarrow \quad \frac{\sin(A - B)}{\sin(A + B)} = \frac{k - 1}{k + 1} \quad \Rightarrow \quad \sin x = \frac{k - 1}{k + 1} \sin \alpha$$

$$10. \quad \text{L.H.S : } 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2} + 2 \cos \frac{A+B+2C}{2} \cdot \cos \frac{A+B}{2}$$

$$= 2 \cos \frac{A+B}{2} \left[\cos \frac{A-B}{2} + \cos \frac{A+B+2C}{2} \right] = 2 \cos \frac{A+B}{2} \left[2 \cos \frac{A+C}{2} \cdot \cos \frac{B+C}{2} \right]$$

$$11. \quad \frac{\sin 7x + \sin 3x}{\cos 7x + \cos 3x} = \tan 5x = \frac{2 \sin \left(\frac{7x+3x}{2} \right) \cos \left(\frac{7x-3x}{2} \right)}{2 \cos \left(\frac{7x+3x}{2} \right) \cos \left(\frac{7x-3x}{2} \right)} = \tan 5x$$

$$12. \quad \frac{1}{\tan 3\alpha - \tan \alpha} + \frac{\tan 3\alpha \tan \alpha}{\tan 3\alpha - \tan \alpha} \Rightarrow \frac{1 + \tan 3\alpha \tan \alpha}{\tan 3\alpha - \tan \alpha} \Rightarrow \frac{1}{\tan(3\alpha - \alpha)} = \frac{1}{\tan 2\alpha} = \cot 2\alpha$$

$$13. \quad \frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C-D)}{\sin(C+D)} \quad \text{Applying 'C' and 'D' : } \frac{\cos(A+B) + \cos(A-B)}{\cos(A+B) - \cos(A-B)} = \frac{\sin(C-D) + \sin(C+D)}{\sin(C-D) - \sin(C+D)}$$

$$\Rightarrow \quad \frac{2 \cos A \cos B}{-2 \sin A \sin B} = \frac{2 \sin C \cos D}{-2 \sin D \cos C} \Rightarrow \cot A \cot B \cot C = \cot D \quad \text{Hence proved.}$$

14. $\frac{\tan^2 A}{1} = \frac{a-b}{a+b} \frac{1}{\tan^2 B}$

$$\Rightarrow \frac{1 - \cos 2A}{1 + \cos 2A} = \frac{(a-b)(1 + \cos 2B)}{(a+b)(1 - \cos 2B)} \quad \Rightarrow \quad \frac{1 - \cos 2A}{1 + \cos 2A} = \frac{a-b + a \cos 2B - b \cos 2B}{a+b - a \cos 2B - b \cos 2B}$$

Apply 'C' and 'D'

$$\Rightarrow \cos 2A = \frac{b - a \cos 2B}{a - b \cos 2B} \quad \text{Put this value of } \cos 2A \text{ in L.H.S.}$$

$$\begin{aligned} \text{L.H.S} &= (a - b \cos 2A)(a - b \cos 2B) = \left[a - b \left(\frac{b - a \cos 2B}{a - b \cos 2B} \right) \right] (a - b \cos 2B) \\ &= a(a - b \cos 2B) - b(b - a \cos 2B) = a^2 - b^2 \end{aligned}$$

15. $\sin x + \sin y = a, \cos x + \cos y = b \Rightarrow \frac{\sin x + \sin y}{\cos x + \cos y} = \frac{a}{b}$

$$\Rightarrow \frac{2 \sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)}{2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)} = \frac{a}{b} \Rightarrow \tan \left(\frac{x+y}{2} \right) = \frac{a}{b} \Rightarrow \sin(x+y) = \frac{2 \tan \left(\frac{x+y}{2} \right)}{1 + \tan^2 \left(\frac{x+y}{2} \right)} = \frac{\frac{2a}{b}}{1 + \frac{a^2}{b^2}} = \frac{2ab}{a^2 + b^2}$$